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Electron Dynamics After Exit Plane of Stationary Plasma Thruster Discharge Chamber

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The paper is devoted to the investigation of electron dynamics in the decaying magnetic field B after the stationary plasma thruster exit plane. It is mainly aimed at the investigation of the electron transfer mechanism in this discharge region. A physico-mathematical model of electron motion is presented in the paper, as well as qualitative discussion of effects resulting in their motion across the field B. Some quantitative assessments for parameters are used for comparing research results with test data. It is shown that anomalous electron transfer after the thruster exit plane is stipulated by the interaction of current of their gradient drift with the field B and has Hall nature. Heat energy of electrons is the driving force of the anomalous transfer, and the power-transfer coefficient does not exceed 0.5. Anomalous transfer is unstable and leads to the excitation of drift plasma oscillations of two types at least: high-frequency magnetohydrodynamic oscillations (∼200 MHz) and low-frequency electromagnetic oscillations (∼20 kHz).

Nomenclature

A = constant factor
a, b, c, d, and a', b', c', d' = vertices of rectangular section for the tube of current
$\mathbf{B}$ = magnetic field inductance
C, C^* = kinetic and electric capacity, respectively
D = side surface area for a tube of current
d = field **B** distribution width after the stationary plasma thruster exit plane
$\mathbf{E}$ = electric field
$\mathbf{E}_{\mathrm{ext}}$ = field of external sources
$\mathbf{E}^{\max}, \mathbf{E}^{\min}, \mathbf{E}^{\mathrm{av}}$ = maximum, minimum, and averaged values of the electric field, respectively
$\mathbf{E}^*$ = induced electric field
$\mathbf{F}_c$ = centrifugal force
$\mathbf{F}_k$ = vector sum of forces acting on the electron
$\mathbf{F}_\nabla$ = ampere force caused by the electron gradient drift
$\mathbf{F}^*_\nabla, \mathbf{F}_{\nabla\Sigma}$ = ampere force resulting from the electron counter-drift $\boldsymbol{v}_{eF}$ and resultant Ampere force, respectively
f_n = frequency of drift magnetohydrodynamic oscillations
$f_{\nabla E}$ = frequency of drift electromagnetic oscillations
H = magnetic field strength
$\mathbf{J}_d, \mathbf{J}_{\mathrm{dr}\Sigma}$ = current caused by diamagnetic effect and total current in the electron drift circuit, respectively
$J_{\mathrm{dr}\Sigma\,\max}$ = maximum total current in the electron drift circuit
$\mathbf{J}_{e\Sigma}$ = resultant electron drift current
$\mathbf{J}_{e\nabla}, \mathbf{J}_{eF}$ = electron gradient drift current and electron drift current in the field **B** under the influence of force $\mathbf{F}_\nabla$, respectively
$\mathbf{J}_\nabla, \mathbf{J}_\perp$ = current of the electrons gradient drift and electron current normal to vector **B**, respectively
$\mathbf{J}^*_{\mathrm{dr}\Sigma}$ = current corresponding to the total drift velocity $\boldsymbol{v}^*_{\mathrm{dr}\Sigma}$
$\mathbf{j}_i, \mathbf{j}_{e\perp}, \mathbf{j}_{ed}, \mathbf{j}_{\mathrm{dr}\Sigma}$ = ion current density, density of electron current normal to vector **B**, diamagnetic current density, and total current density in the electron drift circuit, respectively
k = Boltzmann constant
L = inductance
l, l_{av} = circuit length and averaged circuit length, respectively
m_e, e = electron mass and charge, respectively
n, n_e = plasma density and electron density, respectively
$n^*_e, n_{e\mathrm{av}}$ = local electron density and averaged electron density, respectively
n^*_{e1}, n^*_{e2} = local electron density values
Q = cross-section area encompassed by the current circuit
q_e, q^*_e = compensated and noncompensated electron charge, respectively
$\mathbf{R}$ = radius of curvature for the force line of magnetic field
$\mathbf{r}, \boldsymbol{l}, \mathbf{z}$ = coordinate vectors
S = plasma blob radial cross-section area after the stationary plasma thruster exit plane
T = oscillation period
T_e = electron temperature
t, τ = time and time interval, respectively
t_1, t_2 = time moments
U = potential difference
U_{pz} = potential difference for the electron pressure gradient field
$U_{pz\,\max}$ = maximum potential difference for the electron pressure gradient field
U^*_z = electric field potential difference
W_B, W_P = magnetic field energy and potential energy of electrons, respectively
$W_{B\,\max}, W_{P\,\max}$ = maximum values for the magnetic field energy and potential energy of electrons, respectively
$W_c, W_{c\,\max}$ = potential energy and maximum energy of plasma sheet with noncompensated electron charge, respectively
W_Σ = total energy of plasma sheet

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Δn_e^*	=	local electron density increment
η	=	power transfer ratio
$\sigma_\perp$	=	specific conductivity of plasma across magnetic field
τ_e	=	time interval between electron collisions with ions
$\boldsymbol{v}_{ek}$, $\boldsymbol{v}_{eF}$	=	electron drift velocity in the field **B** caused by force $\mathbf{F}_k$ and by force $\mathbf{F}_\nabla$, respectively
$\boldsymbol{v}_{e\perp}$, $\boldsymbol{v}_e''$	=	electron velocity component directed at a right angle to vector **B** and electron velocity component directed along vector **B**, respectively
$\boldsymbol{v}_{e\nabla}$, $\boldsymbol{v}_{eE}$	=	electron gradient drift velocity in the magnetic field and electron drift velocity in crossed **B** and **E** fields, respectively
$\boldsymbol{v}_{e\nabla\text{av}}$	=	electron drift velocity calculated for the averaged ∇B value
$\boldsymbol{v}_{eE}^*$, $\boldsymbol{v}_{\text{dr}\Sigma}^*$	=	electron drift velocity in field **B** caused by field $\mathbf{E}^*$ and total electron drift velocity, respectively
φ	=	electric potential of plasma
ω	=	circular frequency
ω_c	=	electron cyclotron frequency
ω_{eE}	=	circular frequency for the closed electric drift of electrons
ω_i	=	circular frequency of ion-plasma oscillations
0	=	coordinate center
∇	=	gradient operator
$\nabla P_{e\perp}$	=	electron pressure gradient along the lines normal to vector **B**
$\propto_0$	=	permeability of vacuum

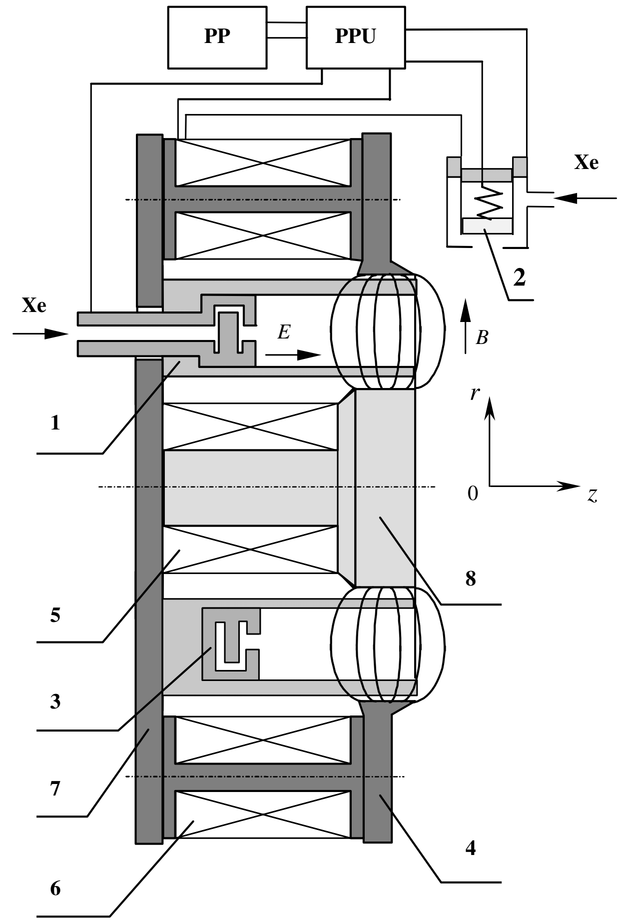


Fig. 1 SPT functional diagram.

I. Introduction

ELECTRON dynamics were studied in the area of the decaying magnetic field after the exit plane of the stationary plasma thruster (SPT).

Plasma formation and acceleration take place in the SPT discharge chamber 1, which comprises the acceleration channel in the form of a cavity closed in the azimuth direction, of ring form as a rule (Fig. 1).

The thruster operating processes develope in the crossed electric **E** and magnetic **B** fields.

The magnetic field **B** in the channel has radial direction **r** mainly and is formed by a magnetic system composed of magnetic coils 5 and 6 and a magnetic conductor with elements 4, 7, 8.

Electric field **E** has longitudinal direction **z**, mainly. It is formed due to the difference of potentials applied to the cathode 2 and anode 3, and its formation is mainly conditioned by the mobility of charged plasma particles across magnetic field **B**.

In the case of the thruster, the schematic diagram of which is shown in Fig. 1, the anode executes the gas distribution function simultaneously, and working gas is fed to the discharge channel through it.

The SPT power supply is provided by the onboard power plant (PP) via the corresponding converters of electric energy, which are usually the parts of the common system securing thruster operation that is called a power processing unit (PPU).

The SPT input parameters, its thrust, specific thrust pulse, and plume divergence, etc., are finally defined by the dynamics of the ion component, and improvement of thruster efficiency is reduced to the development of efficient methods for controlling the processes of ion formation and acceleration.

However, control action is realized for this purpose by forming appropriate distributions for the electron component parameters in the discharge chamber, the theoretical study for which is not finished yet and which restrains the development of physical and engineering fundamentals for the thruster of the next generation to a considerable degree.

In particular, there is no clear understanding of some mechanisms of electron transfer across magnetic field and, in this connection, of the conditions for the formation of electric field distribution in the discharge chamber.

Electron dynamics after the discharge chamber exit plane are studied to the least degree. Here, the value of magnetic inductance **B** drops ($\partial B/\partial z < 0$) from some maximum value $\mathbf{B}_{\max}$ in the exit plane area and approaches zero at a large enough distance from it (Fig. 2).

Ion acceleration is mainly finished after the exit plane, and their velocity of ordered motion here is higher on average than their velocity within the discharge chamber. As a result of this, ion density after the exit plane does not exceed its values in the acceleration zone.

Propellant acceleration after the exit plane is mainly finished as well, and atom density here is substantially less than in the discharge chamber.

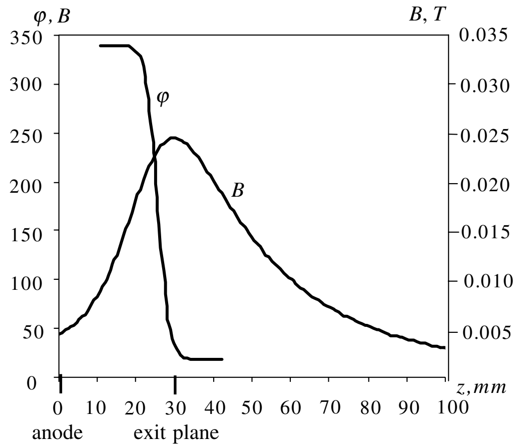
Thereby, the role of the effects of electron scattering on heavy particles in electron transfer across a magnetic field after the exit plane is less than the role in the transfer taking place in the acceleration zone.

The role of the near-wall conductivity in the area under consideration is reduced to minimum as well because of the large enough distance between the structural elements and the plasma blob core.

Nevertheless, a weak electric field after the exit plane in comparison to its value in the discharge chamber accelerating channel (refer, in particular, to the distribution of plasma electric potential φ in Fig. 2) witnesses the existence of high electron mobility across the magnetic field here.

Such mobility is called "anomalous" because it is not explained by classical mechanisms of electron transfer in plasma retained by the magnetic field.

In the known works carried out at the I. V. Kurchatov Institute of Atomic Energy, the appearance of anomalous mobility was related to plasma oscillations, the intensity of which, in the SPT, grows substantially at $\partial B/\partial z < 0$ [1–3].



B – calculated values along the line equidistant from the channel walls;

φ - values obtained by test along the outer wall

Fig. 2 Distributions of magnetic field B and plasma potential φ in the SPT-100 at the discharge voltage of 350 V.

The common condition at which the stability of processes in plasma breaks was obtained by Yesiptchuk and Tilinin in a form of criterion [3]

$$\frac{\partial}{\partial z}\left(\frac{H}{n}\right) < 0 \tag{1}$$

where H is the intensity of magnetic field and n is the plasma density.

According to the results these works, the appearance of anomalous effects after the SPT exit plane, where sufficiency of stabilizing factors is not obvious, may be a consequence of wave process development, a precondition of which is the magnetic inductance drop.

But mechanisms of excited oscillations, in view of which conditions for the appearance of anomalous plasma mobility may be considered, as well as methods for stabilizing and controlling currents associated with it are not studied in detail in the known publications.

Results of test studies for SPT performance, obtained by the Orleans University in France, supplement the overall picture of processes in a thruster to a great extent [4]. But their analysis is not finished yet and does not reveal mechanisms of the investigated processes evolution also.

This paper is devoted to the investigation of the dynamics of the plasma electron component in the decaying magnetic field after the SPT exit plane in view of the processes of electron transfer from the near-cathode area to the discharge chamber.

It is shown that plasma oscillations being excited after the thruster exit plane are not the reason but are the result of the processes of electron transfer across the decaying magnetic field.

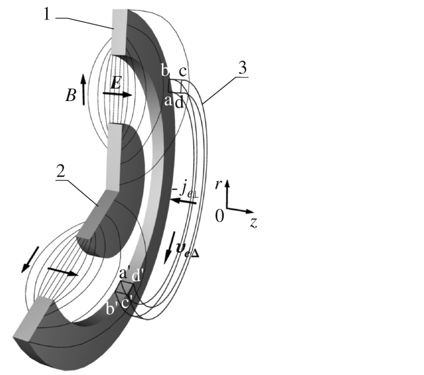
II. Physical and Mathematical Model Used for Study

Let us study by examining the equilibrium state of the electron in some elementary current tube closed in the azimuth direction that is chosen in the plasma blob after the SPT exit plane (Fig. 3) [5] in the drift approximation.

The mentioned current tube is cut by two radial planes in Fig. 3 (sections $abcd$ and $a'b'c'd'$).

The tube is in the electric field **E**, which is directed along the **z** axis. The flow of accelerated ions characterized by the ion density $\mathbf{j}_i$ moves in the same direction.

The model assumes also that the tube is crossed by magnetic field **B** ($\partial B/\partial z < 0$) that has radial direction **r** mainly and decays along **z**, the lines of force of which lie in the planes forming radial cross sections of the thruster (plane zor in Fig. 4).



1 – outer magnetic pole; 2 – inner magnetic pole; 3 – tube of the electron drift current

Fig. 3 Magnetohydrodynamic electron transfer in the decaying magnetic field B.

The drift approximation limits the area of study formally by the condition $\omega_c \tau_e \gg 1$, where ω_c is the electron cyclotron frequency and τ_e is the time interval between electron collisions with ions.

This approximation, as the idealized representation of the charged particle motion in the perfectly conducting medium, brings the plasmadynamic system to the level at which it is considered adiabatic [6]. In the drift approximation, true particle motion that is associated with the variation of its energy is limited by the Langmuir circumference, and the center motion is considered the motion of the particle with constant drift velocity.

But the interpretation of plasma as of a perfectly conducting medium leads to formal difficulties in the system state analysis, and so let us consider that at the high enough values of plasma conductivity, it nevertheless has some limitations in value.

In the model under consideration, these limitations refer to the ordered motion of electrons only, the current $\mathbf{j}_{e\perp}$ conditioned by their anomalous transfer across the field **B** being such.

As to the electron drift in the magnetic field, let us consider (not deviating from the formal conditions of appropriateness of the drift approximation use) that the drift takes place in the perfectly conducting medium.

Gradient drift of electrons in the azimuth direction takes place in the tube under the influence of energy of their heat motion in the magnetized plasma under the conditions of decaying field **B**. Velocity of this gradient drift is defined by the following expression:

$$\boldsymbol{\upsilon}_{e\nabla} = \frac{kT_e}{e}\frac{[\mathbf{B}\cdot\nabla B]}{B^3} \tag{2}$$

where T_e is the electron temperature, k is the Boltzmann constant, and e is the electron charge.

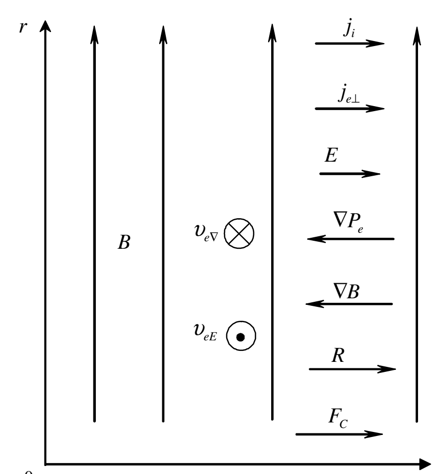


Fig. 4 Vector model for the electron motion in the decaying magnetic field.

Azimuth current $\mathbf{J}_\nabla$ is the consequence of electron drift. Density of this current is defined by the following expression:

$$\mathbf{j}_{e\nabla} = -en_e \boldsymbol{v}_{e\nabla} \tag{3}$$

where n_e is the electron density.

The following ampere force acts upon the electron drifting at the velocity $\boldsymbol{v}_{e\nabla}$ in the field $\mathbf{B}$:

$$F_\nabla = -e[\boldsymbol{v}_{e\nabla} \cdot \mathbf{B}] \tag{4}$$

Electrons drifting inside the tube are drawn into the region of strong field $\mathbf{B}$ under the influence of force $\mathbf{F}_\nabla$ that results in the electric current $\mathbf{J}_\perp$ in it, with the direction $\mathbf{z}$ and density $\mathbf{j}_{e\perp}$.

The following expression may be written for current $\mathbf{J}_\perp$:

$$\mathbf{J}_\perp = \int_A \mathbf{j}_{e\perp}\,\mathrm{d}A \tag{5}$$

where A is the area of the tube side surface.

However, dynamics of the drifting electron pulling ase somewhat different from the observed in the case of solid conductor with current in the magnetic field.

The matter is that the force $\mathbf{F}_\nabla$ causes electron drift with the following velocity:

$$\boldsymbol{v}_{eF} = -\frac{\mathbf{F}_\nabla \cdot \mathbf{B}}{e \cdot B^2} \tag{6}$$

and thus the appearance of the oppositely directed force is:

$$\mathbf{F}^* = -e[\boldsymbol{v}_{eF} \cdot \mathbf{B}] \tag{7}$$

Dynamic equilibrium will be established between forces $\mathbf{F}_\nabla$ and $\mathbf{F}^*_\nabla$.

To explain this, let us examine the dynamics of current being the result of the gradient drift of electrons.

Its resultant value attributed to a single electron may be written as

$$\mathbf{J}_{e\Sigma}(t) = \mathbf{J}_{e\nabla} - \mathbf{J}_{eF}(t) \tag{8}$$

where $\mathbf{J}_{e\nabla}$ is the gradient drift current attributed to a single electron, and $\mathbf{J}_{eF}(t)$ is the current of drift, conditioned by the action of force $\mathbf{F}^*_\nabla$, which is attributed to a single electron.

The value of current $\mathbf{J}_{eF}(t)$ is the function of time t, because this current is inductively coupled with current $\mathbf{J}_{e\nabla}$ and is directed toward the latter. Because of this, their sum $\mathbf{J}_{e\Sigma}(t)$ will vary from a maximum value that is equal to $\mathbf{J}_{e\nabla}$ down to zero at the current equality.

If one imagines that the drift loop is a coil with one turn, and electron movement along $\mathbf{z}$ takes place in the plasma sheet with capacitance C, the dynamics of current $\mathbf{J}_{e\Sigma}(t)$ variation may be described by the following equation:

$$L\frac{\mathrm{d}\mathbf{J}_{e\Sigma}(t)}{\mathrm{d}t} = -U(t) \tag{9}$$

where $U(t)$ is the potential difference formed in the lateral direction of current tube, and L is the inductance of current loop.

The solution of Eq. (9) shows that harmonic oscillations of resultant current $\mathbf{J}_{e\Sigma}(t)$ occur in the electron drift circuit. The resultant force acting upon the electron is oscillating together with current is:

$$\mathbf{F}_{\nabla\Sigma}(t) = \mathbf{F}^*_\nabla(t) + \mathbf{F}_\nabla = \mathbf{J}_{e\Sigma}(t) \cdot \mathbf{B} \tag{10}$$

As at the oscillations the resultant current $\mathbf{J}_{e\Sigma}(t)$ do not pass through zero value, the following inequation is satisfied always:

$$\int_T \mathbf{J}_{e\Sigma}(t)\,\mathrm{d}t > 0 \tag{11}$$

where T is the oscillation period.

The inequality of the integral current value $\mathbf{J}_\perp$ to zero is the consequence of Eq. (11):

$$\int_T \mathbf{J}_\perp(t)\,\mathrm{d}t > 0 \tag{12}$$

The power-transfer ratio of the discussed plasmadynamic system may be defined by the following expression:

$$\eta = \frac{\int_T \mathbf{F}_{\nabla\Sigma}(t)\,\mathrm{d}t}{\mathbf{F}_\nabla \cdot T} \tag{13}$$

In the matched transfer mode, $\eta = 0.5$.

Thus, anomalous electron transfer across the decaying magnetic field after the SPT exit plane is conditioned by the interaction of current caused by the gradient drift of electrons with this field of current.

Under real conditions, the value

$$\int_T \mathbf{J}_\perp(t)\,\mathrm{d}t$$

depends on some set of forces acting on electrons. Let us examine them.

Under the influence of fields $\mathbf{E}$ and $\mathbf{B}$, an electron drift takes place in the tube, the velocity of which is defined by the following expression:

$$\boldsymbol{v}_{eE} = \frac{\mathbf{E} \cdot \mathbf{B}}{B^2} \tag{14}$$

Such drift causes origination of ampere force $-e[\boldsymbol{v}_{eE} \cdot \mathbf{B}]$ acting upon electrons, which has the same value as the force of electric field, but in the opposite direction.

Current appearing at the anomalous transfer leads to the emergence of friction force $e\mathbf{j}_{e\perp}/\sigma_\perp$, where $\sigma_\perp$ is the specific conductivity of plasma in the transverse direction.

The induced electric field $\mathbf{E}^*$, as the component of field $\mathbf{E}$, will be the result of the action of this force.

Among other forces that should be taken into account for the analysis of electron dynamics, there are the following: force $-(\nabla P_{e\perp}/n_e)$, acting on the electrons across the magnetic field and caused by the potential field of the electron pressure gradient $\nabla P_{e\perp}$, and centrifugal force

$$\mathbf{F}_c = \frac{m_e v^2_{e//}}{\mathbf{R}} \tag{15}$$

acting on the electron at its motion along the force line, where m_e is the electron mass, $\boldsymbol{v}_{e//}$ is the electron velocity component directed along the force line of magnetic field, and $\mathbf{R}$ is the radius of curvature for the force line of magnetic field.

Forces $-(\nabla P_{e\perp}/n_e)$, $(m_e v^2_{e//})/R$, and $e\mathbf{j}_{e\perp}/\sigma_\perp$ after the SPT exit plane coincide in direction with each other and are directed toward the force $-e[\boldsymbol{v}_{e\nabla} \cdot \mathbf{B}]$.

The vector sum of forces

$$\mathbf{F}_k = -\frac{\nabla P_{e\perp}}{n_e} + \frac{m_e \cdot v^2_{e//}}{\mathbf{R}} - e[\boldsymbol{v}_{e\nabla} \cdot \mathbf{B}] \tag{16}$$

causes electron drift in the magnetic field, the velocity of which is defined by the following expression:

$$\boldsymbol{v}_{ek} = -\frac{\mathbf{F}_k \cdot \mathbf{B}}{eB^2} \tag{17}$$

Interaction of current, caused by the mentioned drift, with field $\mathbf{B}$ leads to the occurrence of ampere force acting on the electrons:

$$\mathbf{F}^*_k = -e[\boldsymbol{v}_{ek} \cdot \mathbf{B}] \tag{18}$$

which is opposite to the direction of the vector sum of forces $\mathbf{F}_k$.

As is obvious from expressions (17) and (18), there is a feedback between forces $\mathbf{F}_k$ and $\mathbf{F}^*_k$. This feedback is effected by drift velocity $\boldsymbol{v}_{ek}$ and causes harmonic oscillations of the force $\mathbf{F}^*_k$.

In view of the analysis for forces acting on the electron, let us write equation for its motion in the decaying magnetic field after the SPT exit plane.

In the quasi-hydrodynamic approximation, this equation will have the following form:

$$m_e \frac{\mathrm{d}\boldsymbol{v}_\perp(t)}{\mathrm{d}t} = -e\mathbf{E} - e[\boldsymbol{v}_{eE}\cdot\mathbf{B}] - e[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] - \frac{\nabla P_{e\perp}}{n_e} + \frac{m_e v_{//}^2}{\mathbf{R}} + \frac{e\,\mathbf{j}_{e\perp}(t)}{\sigma_\perp} - e[\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}] \tag{19}$$

where $\boldsymbol{v}_\perp(t)$ is the rate of anomalous electron transfer. Let us study the conditions under which anomalous electron transfer across decaying magnetic field is realized, on the basis of Eq. (19).

III. Analysis for Factors Conditioning Anomalous Electron Transfer Across Magnetic Field

Let us make preliminary analysis under the assumption that the force lines of the magnetic field have some small curvature ($\mathbf{R}\to\infty$). This allows simplification for the model without distorting the qualitative pattern of processes and, in particular, neglecting for the centrifugal force F_c acting on the electron during its motion along the force line.

In view of the smallness of m_e and obvious equality $e\mathbf{E} = -e[\boldsymbol{v}_{eE}\cdot\mathbf{B}]$, expression (19) may be written in the following form:

$$\frac{e\,\mathbf{j}_{e\perp}(t)}{\sigma_\perp} = e[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] + \frac{\nabla P_{e\perp}}{n_e} + e[\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}] \tag{20}$$

Based on Eq. (20), we can write the expression for the electron current conditioned by the anomalous electron transfer as

$$\mathbf{j}_{e\perp}(t) = \sigma_\perp\left([\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] + \frac{\nabla P_{e\perp}}{en_e} + [\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}]\right) \tag{21}$$

In this equation, expression $[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}]$ characterizes the electron transfer driving force, and expression

$$\frac{\nabla P_{e\perp}}{en_e} + [\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}]$$

is its constraint.

Let us consider the case

$$[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] = -\frac{\nabla P_{e\perp}}{en_e} \tag{22}$$

Equation (22) means that the potential field $[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}]$ characterizing the electron transfer driving force is equilibrated by the field $-(\nabla P_{e\perp}/en_e)$. Though these fields are self-consistent, there is no direct inductive coupling between them.

It is not difficult to make oneself sure that under the condition of Eq. (22) the equation $[\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}] = 0$ is satisfied also. Thus, it may be written formally under this condition:

$$\mathbf{j}_{e\perp} = 0 \tag{23}$$

Expression (23) is obviously true also at

$$[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] < -\frac{\nabla P_{e\perp}}{en_e} \tag{24}$$

that is, under the conditions in Eqs. (22) and (24), the region of decaying magnetic field after the SPT exit plane is blocked for electrons.

Let us consider the following case:

$$[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}] > -\frac{\nabla P_{e\perp}}{en_e} \tag{25}$$

Inequality (25) means that the field $-(\nabla P_{e\perp}/en_e)$ in its value does not secure an equilibrium state for an electron at the force line, and, as follows from Eq. (21), it is reached due to the force $e[\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}]$ appearing with this.

As there is inductive coupling between fields $[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}]$ and $[\boldsymbol{v}_{ek}(t)\cdot\mathbf{B}]$, and the equilibrium state of electron component is formed at the dynamic mode, then, as it follows from conclusions made in Sec.II, the integral of the transferred current at any time interval τ is not equal to zero:

$$\int_\tau \mathbf{j}_{e\perp}(t)\,\mathrm{d}t > 0 \tag{26}$$

In view of the fact that negative values for the gradients of field **B** and density n_e are realized after the SPT exit plane ($\partial B/\partial z < 0$ and $\partial n_e/\partial z < 0$), condition (25) may be written in the following form:

$$\frac{1}{n_e}\left|\frac{\partial n_e}{\partial z}\right| < \frac{1}{B}\left|\frac{\partial B}{\partial z}\right| \tag{27}$$

Thus, at $\partial B/\partial z < 0$, the transfer of electrons across the magnetic field after the SPT exit plane takes place when the absolute value of the relative magnetic inductance gradient exceeds the absolute value of the relative electron density gradient.

The physical model, within the frames of which the study was made, assumes some small curvature for the lines of force ($\mathbf{R}\to\infty$). In real devices, the radius value is limited, and the motion of electron along the force line leads to the occurrence of centrifugal force $\mathbf{F}_C$ acting on it.

For magnetic fields with the configuration of force lines that are typical for the region adjacent to the SPT exit plane ($\partial B/\partial z < 0$ and $\partial R/\partial z < 0$), the force $\mathbf{F}_C$ is directed toward the force $-e[\boldsymbol{v}_{e\nabla}\cdot\mathbf{B}]$, and at $\mathbf{E} = 0$, it leads to the reduction of value $\mathbf{j}_{e\perp}$.

In the extreme case that might be realized in the magnetic field of the conductor with current, for example, where $R\sim 1/B$, the equality of forces occurs

$$-\mathbf{F}_C = \mathbf{F}_\nabla \tag{28}$$

and the magnetohydrodynamic transfer of electrons stops.

The following may be written for the magnetic fields existing in real devices:

$$-\mathbf{F}_\nabla > \mathbf{F}_C \neq 0 \tag{29}$$

In view of Eq. (29), expression (27) needs to be clarified.

In particular, analysis for the $\mathbf{F}_C$ influence upon electron transfer may be made at the introduction of factor A into Eq. (27):

$$\frac{1}{n_e}\left|\frac{\partial n_e}{\partial z}\right| < A\frac{1}{B}\left|\frac{\partial B}{\partial z}\right| \tag{30}$$

The factor values are within the range $0 < A < 1$:

$$A = 0 \quad \text{at} \quad R\sim\frac{1}{B} \qquad A\to 1 \quad \text{at} \quad R\to\infty$$

The availability of electric field $\mathbf{E} > 0$ causes factor A growth, because the force acting on electrons in this field equalizes centrifugal force $\mathbf{F}_C$ partially or completely.

At $\mathbf{E} \geq (\mathbf{F}_C/e)$, its value is equal to the limit value ($A = 1$).

Thus, conversion of the electron heat motion energy into the energy of ordered motion of their flow, moving from the near-cathode region to the discharge chamber across the magnetic field, takes place in the SPT discharge region located after its exit plane in the decaying magnetic field. The plasmadynamic electron transfer taking place with this is a consequence of the plasma electron component flow that is caused by the gradient drift of electrons and has Hall nature.

Results of the analysis allow the definition of some ways to control the value of electron current $\mathbf{J}_\perp$ in the SPT discharge that is stipulated by plasmadynamic electron transfer after the exit plane.

One of them is realized by creating a buffer zone with reduced or zero gradient **B** at the discharge chamber inlet, as shown in Fig. 2, due to which electron accumulation takes place in the mentioned zone. The density n_e gradient growth is secured thereby, and optimum value for the difference of values being the parts of inequality (30) may be reached:

$$\delta_{\text{opt}} = A\frac{1}{B}\left|\frac{\partial B}{\partial z}\right| - \frac{1}{n_e}\left|\frac{\partial n_e}{\partial z}\right| \tag{31}$$

However, at the current $\mathbf{J}_\perp$ proportioning realized by regulating the value $\partial n_e/\partial z$, its instability may be observed because, in this case, the force $-(\nabla P_{e\perp}/n_e)$ that counters transfer is stipulated by this transfer and depends on its value.

Another method supposes formation of the magnetic field with increased curvature of force lines after the discharge chamber exit plane, in which proportioning of the $\mathbf{J}_\perp$ value may be secured owing to the centrifugal force $\mathbf{F}_C$ acting on electrons at their motion along the force lines of the magnetic field. At such proportioning, the stability of electron transfer across the magnetic field is increased because, in this case, the force $\mathbf{F}_C$ counteracting the transfer does not depend on the $\mathbf{J}_\perp$ value.

IV. Analysis for Stability of Plasmadynamic Electron Transfer After Exit Plane of Stationary Plasma Thruster

A. Study for Characteristics of Drift Magnetohydrodynamic Plasma Oscillations at $\partial B/\partial z < 0$

In the general case, analysis for the stability of plasmadynamic electron transfer may be made in view of availability of at least two potential fields inserted into each other in plasma.

One of the fields is electric field **E** and another is the field of the electron pressure gradient $-(\nabla P_e/en_e)$. The potential energy of electrons changes at their movement in the first as well as in the second field.

In the problem under consideration at $\mathbf{E} \approx 0$, this change is associated with the overcoming of force $-(\nabla P_{e\perp}/n_e)$ that counteracts transfer. The appearance of the mentioned force is a result of the electron density n_e growth in the buffer zone, where the magnetic field gradient is reduced in its absolute value and is equal to zero.

In the quasi-hydrodynamic approximation, the force $-(\nabla P_{e\perp}/n_e)$ acts on the electrons by means of diamagnetic current, the density of which may be defined in accordance with the following expression:

$$\mathbf{j}_{ed}(t) = kT_e\frac{\mathbf{B}\cdot\nabla n_e(t)}{B^2} \tag{32}$$

In view of this, the following expression may be written for the total current density in the electron drift loop:

$$\mathbf{j}_{\text{dr}\Sigma}(t) = \mathbf{j}_{e\nabla} - \mathbf{j}_{ed}(t) \tag{33}$$

where $\mathbf{j}_{e\nabla}$ is the current density for the gradient electron drift ($\mathbf{j}_{e\nabla} = -en_e\boldsymbol{v}_{e\nabla}$).

In accordance with Eq. (33), it is possible to write the expression for the total current in the electron drift loop after the SPT exit plane:

$$\mathbf{j}_{\text{dr}\Sigma}(t) = \mathbf{j}_\nabla - \mathbf{j}_d(t) \tag{34}$$

where $\mathbf{J}_\nabla$ is the total current resulting from the gradient electron drift, and $\mathbf{J}_d(t)$ is the total current caused by diamagnetic effect in the electron drift loop.

In view of Eq. (9), the dynamics of current $\mathbf{J}_{\text{dr}\Sigma}(t)$ variation may be described by the following equation:

$$L\frac{\text{d}J_{\text{dr}\Sigma}(t)}{\text{d}t} = -U_{pz}(t) \tag{35}$$

where L is the inductance of the current loop, and $U_{pz}(t)$ is the potential difference formed at the length of the electron transfer in the field of electron pressure gradient.

The following simplified expression may be written for $U_{pz}(t)$ at $\mathbf{R}\to\infty$:

$$U_{pz}(t) = \frac{kT_e}{e}\int_o^d \frac{1}{n_e(t)}\frac{\partial n_e(t)}{\partial z}\,\text{d}z \tag{36}$$

where d is the distance at which the magnetic field **B** variation takes place from maximum values $B_{\max}$ down to the values at which it does not influence the electron motion.

Expression (35) may be written in the form convenient for analysis:

$$L\frac{\text{d}q_e^2(t)}{\text{d}t^2} = -\frac{q_e(t)}{C} \tag{37}$$

where q_e is the electron charge in the current loop, and C is the coefficient having the dimensions of capacitance.

As is well known, Eq. (37) describes simple harmonic oscillations of charge with the frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{38}$$

Let us assess its value.

At the oscillations, the magnetic field energy W_B converts to the potential energy W_P of electrons coming to the buffer zone located near the exit plane in the region with maximum values of magnetic inductance.

Neglecting losses, we can write

$$W_{B\max} = W_{P\max} = \frac{LJ^2_{\text{dr}\Sigma\max}}{2} = \frac{CU^2_{pz\max}}{2} \tag{39}$$

Whence

$$LC = \frac{L^2J^2_{\text{dr}\Sigma\max}}{U^2_{pz\max}} \tag{40}$$

In view of Eq. (40), we obtain the following expression for frequency:

$$f_n = \frac{1}{2\pi}\frac{U_{pz\max}}{LJ_{dr\Sigma\max}} \tag{41}$$

Let us assess values of parameters being the parts of expression (41).

$U_{pz\max}$ may be defined from Eq. (36) under the condition

$$\frac{1}{n_e}\left|\frac{\partial n_e}{\partial z}\right| = \frac{1}{B}\left|\frac{\partial B}{\partial z}\right| \tag{42}$$

Using distribution $\mathbf{B}(z)$ in Fig. 2 for $kT_e/e = 10$ V, we obtain

$$U_{pz\max} \approx \frac{2kT_e}{e} \approx 20 \text{ V} \tag{43}$$

In accordance with expression (34), the maximum value for current $\mathbf{J}_{\text{dr}\Sigma}(t)$ is reached at $\mathbf{J}_d(t) = 0$. Then

$$J_{\text{dr}\Sigma\max} = J_\nabla = -e\int_s n_e\upsilon_{e\nabla}\,\text{d}s = kT_e\int_s \frac{n_e}{B^2}\frac{\partial B}{\partial z}\,\text{d}s \tag{44}$$

where S is the area of plasma formation radial section after the SPT exit plane.

$J_{\text{dr}\Sigma\max}$ may be assessed also at $n_e \approx 2\times 10^{17}$ m^{-3} and $kT_e/e \approx 10$ V, using distribution $B(z)$ in Fig. 2.

As a result of this assessment, we obtain

$$J_{\text{dr}\Sigma\max} \approx 0.5 \text{ A} \tag{45}$$

The value of ringlike loop inductance may be defined by the expression

$$L = \frac{\propto_0 Q}{l} \tag{46}$$

where Q is the area of the section encompassed by the current loop, l is the current loop length, and $\propto_0$ is the magnetic constant.

In particular, for SPT-100, its value is

$$L \approx 3 \times 10^{-8}\ \text{H} \quad (47)$$

Using results of assessments for the parameters being the parts of expression (41) [refer to expressions (43), (45), and (47)], we obtain

$$f_n \approx 2.0 \times 10^8\ \text{Hz} \quad (48)$$

The considered drift magnetohydrodynamic oscillations of plasma are quasi-longitudinal oscillations of density n_e because diamagnetic current $\mathbf{j}_{ed}$, having azimuthal direction by which the action of force $-(\nabla P_{e\perp}/n_e)$ is realized, does not lead to the variation of electron drift velocity.

The linear scale of such oscillations depends on a number of factors and, in the general case, plasma formation after the SPT exit plane may be represented by a number of narrow, sequentially located sheets in which they take place.

At the uniform distributions of $(\nabla B/B)(z)$ and $T_e(z)$ in the sheet, the statement of quasi-longitudiallity becomes exact.

Existence of oscillations in the SPT discharge with the frequency of about several hundred MHz is presented by Tilinin in the article published based on the results of experimental work fulfilled at the I. V. Kurchatov Institute of Atomic Energy [7].

B. Study for Characteristics of Drift Electromagnetic Oscillations of Plasma at $\partial B/\partial z < 0$

In the region of maximum values for $\mathbf{B}$, the magnetohydrodynamic effect stipulating electron transfer becomes weaker because of the reduction of the gradient drift velocity; this is why the electric field $\mathbf{E}$ is found in this region.

Under the action of electric field $\mathbf{E}$, electrons become involved in the azimuthal motion with circular frequency

$$\omega_{eE} = \frac{2\pi \upsilon_{eE}}{l} = \frac{2\pi E}{lB} \quad (49)$$

According to assessments, the ω_{eE} value is higher than the frequency of ion-plasma oscillations ω_i in the region under consideration ($\omega_i < \omega_{eE}$). That is why there are conditions in the circuit for the development of low-frequency plasma oscillations related to the drift instability of frequency ω, which has no limitations from below.

The mentioned limitations are raised because low-frequency drift oscillations are developed at the carrier frequency ω_{eE}, modulating its amplitude.

Let us show the mechanism of such oscillations development.

As a result of anomalous electron transfer, the charge separation takes place in such a way that the induced electric field $\mathbf{E}^*$ appearing with this compensates the field of external sources $\mathbf{E}_{\text{ext}}$ in part or in full; the following expression may be written for the total electric field $\mathbf{E}$ in this case:

$$\mathbf{E} = \mathbf{E}_{\text{ext}} + \mathbf{E}^* \geq 0 \quad (50)$$

At full compensation ($\mathbf{E} = 0$), electric drift of electrons disappears, but their gradient drift remains inalterable; therefore, their transfer across magnetic field remains as well.

But further electron transfer may only take place if it is accompanied by some back ion current $\mathbf{j}_i < 0$ supporting the equilibrium state of plasma at $\mathbf{E} = 0$. And, as follows from conclusions of Sec. III, this transfer will proceed until the force $\mathbf{F}_\nabla = -e[\boldsymbol{\upsilon}_{e\nabla} \cdot \mathbf{B}]$ causing it is equilibrated by the counteracting force $-(\nabla P_{e\perp}/n_e)$.

Variation of direction for the electric field $\mathbf{E}$ does not take place in this case because it would cause growth of the back ion current $\mathbf{j}_i$, as a result of which the equilibrium satisfying condition (50) would be recovered.[†]

Let us assume now that, in the plasma loop of current caused by electron drift, some local increase in their density takes place: $n_e^* > n_{e\text{av}}$, where $n_{e\text{av}}$ is the electron density averaged over the loop length (refer to the system state for time moment t_1 that is shown conditionally in Fig. 5).

Nevertheless, at the mentioned conditions for the ion component, the tube of current caused by electron drift may be considered as uniform with density $n_{e\text{av}}$ being uniformly distributed in the azimuth direction, and on average, the condition (50) will be met in each cross section.

However, at the loop part, at which the electron density is increased ($n_e^* = n_{e\text{av}} + \Delta n_e^*$), the total electric field $\mathbf{E}$ will become negative, and, because of this, counteract the magnetohydrodynamic electron transfer.

This is illustrated by Fig. 6, in which electric field distributions along $\mathbf{z}$ after the SPT exit plane are shown: $\mathbf{E}^{\min}(z)$ is the distribution in the current loop cross section at its part characterized by high electron density $n_e^* = n_{e\text{av}} + \Delta n_e^*$, $\mathbf{E}^{\max}(z)$ is the distribution in the loop cross sections of its rest part, in which distribution is rather uniform, and $\mathbf{E}^{\text{av}}(z)$ is the distribution averaged over the loop length l.

In the quasi-hydrodynamic approximation, the counteraction to electron transfer is related to the occurrence of their electric drift, the velocity of which

$$\boldsymbol{\upsilon}^*_{eE}(t) = \frac{\mathbf{E}^*(t) \cdot \mathbf{B}}{B^2} \quad (51)$$

is directed toward the gradient drift velocity $\boldsymbol{\upsilon}_{e\nabla}$.

In this case, expression for the total drift velocity will have the following form:

$$\boldsymbol{\upsilon}^*_{\text{dr}\Sigma}(t) = \boldsymbol{\upsilon}_{e\nabla} - \boldsymbol{\upsilon}^*_{eE}(t) \quad (52)$$

The following current corresponds to it:

$$\mathbf{J}^*_{\text{dr}\Sigma}(t) = -e \int_s n_e(t) \boldsymbol{\upsilon}^*_{\text{dr}\Sigma}(t)\, \text{d}s \quad (53)$$

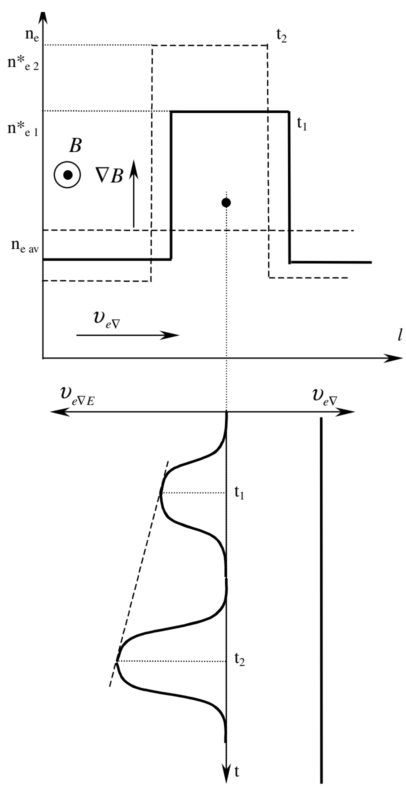


Fig. 5 Electron "spoke" formation at $\partial B/\partial z < 0$.

[†]This statement is related to the model simplification with regard to the dynamics of the ion component.

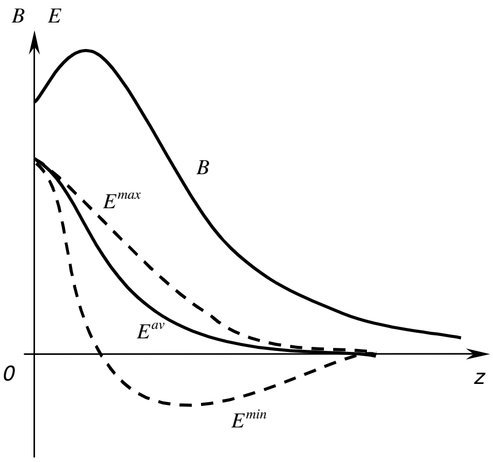


Fig. 6 Formation of electric field distributions after the exit plane of stationary plasma thruster at the nonuniform azimuthal distribution of electron density.

Self-inductance electromotive force at the current $\mathbf{J}^*_{\mathrm{dr}\Sigma}(t)$ variation in the ringlike loop will be obviously equal to the potential difference $U^*_z(t)$ conditioned by charge separation along z in the plasma sheet that forms this loop.

$$L\frac{\mathrm{d}J^*_{\mathrm{dr}\Sigma}(t)}{\mathrm{d}t} = -U^*_z(t) \tag{54}$$

or

$$L\frac{\mathrm{d}^2 q^*_e(t)}{\mathrm{d}t^2} = -\frac{q^*_e(t)}{C^*} \tag{55}$$

where q^*_e is the charge of the electrons that shifted during transfer, and C^* is the capacitance of the sheet in which the charge separation takes place.

The solution of Eq. (55) indicates that at the loop part with high electron density simple harmonic oscillations with the following frequency will take place:

$$f_{\nabla E} = \frac{1}{2\pi}\sqrt{\frac{1}{LC^*}} \tag{56}$$

During the process of such oscillations, any azimuthal nonuniformity of electron density will become stronger because the value of induced electric field $\mathbf{E}^*$, and thus the velocity of electron drift $\boldsymbol{v}^*_{eE}(t)$ caused by it, depend on the electron density, and because of this, during the electron drift process, the region with high electron density will move up to the adjacent region characterized by lower density, and their densities will be summed up (see system state at t_2 in Fig. 5). As a result, the whole plasma will be collected in one blob (spoke), inside which low-frequency oscillations will take place.

Let us assess their frequency.

At the oscillations, the magnetic field energy

$$W_B = \frac{LJ^{*2}_{\mathrm{dr}\Sigma}}{2}$$

converts into the potential energy of plasma sheet as of the ringlike capacitor

$$W_C = \frac{q^{*2}_e}{2C}$$

The total energy W_Σ of plasma system during oscillations remains constant:

$$W_\Sigma = W_B + W_C = W_{B\,\max} = W_{C\,\max} \tag{57}$$

Let us write taking expressions (52) and (53) into account:

$$LC^* = \frac{q^{*2}_{e\,\max}}{J^2_\nabla} \tag{58}$$

So

$$f_{\nabla E} = \frac{1}{2\pi}\frac{J_\nabla}{q^*_{e\,\max}} = \frac{-e\int_s n_e \upsilon_{e\nabla}\,\mathrm{d}s}{-2\pi e\int_s l\cdot n_e\,\mathrm{d}s} \tag{59}$$

The following expression may be used for assessing $f_{\nabla E}$ value:

$$f_{\nabla E} \approx \frac{\upsilon_{e\nabla\mathrm{av}}}{2\pi l_{\mathrm{av}}} \tag{60}$$

where $\upsilon_{e\nabla\mathrm{av}}$ is the gradient electron drift velocity calculated for an averaged value ∇B, and l_{cp} is the averaged length for the current loop (electron drift loop after the SPT exit plane).

In particular, for SPT-100 ($\upsilon_{e\nabla\mathrm{av}} \approx 3\times 10^4$ m/s, $l_{\mathrm{av}} \approx 0.08\pi$ m), we obtain

$$f_{\nabla E} \approx 2\times 10^4\ \mathrm{Hz} \tag{61}$$

Low-frequency modulation of electron current transferred to the discharge chamber causes modulation at the same frequency for the processes of ion formation and acceleration, as a result of which plasma acceleration in SPT may have a pulsed pattern.

In particular, the existence of low-frequency modulation of ion current in SPT is indicated in [8].

Thus, results of analysis for the stability of plasmadrodynamic electron transfer after the SPT exit plane allow to state that stabilization of electron current $\mathbf{J}_\perp$ at the thruster discharge chamber inlet may be realized at two modes at least, that is, at high frequency (~200 MHz) and low frequency (~20 kHz) modes.

As follows from Eq. (37) and (55), for a loop with inductance L, the dynamics of current stabilization modes depend on the values of parameters C and C^*, which, in particular, define current $\mathbf{J}_\perp$ oscillation amplitudes at the considered modes.

In contrast to capacitance C^* that characterizes accumulation of charges q in the plasma medium, the capacitance C has a kinetic nature and characterizes accumulation of electron gas with density n_e in this system.

Using expressions (38) and (56), it is possible to obtain relationship for the values of electric C^* and kinetic C capacitances characterizing plasma system under consideration:

$$C^* = 10^8 C \tag{62}$$

from which it follows that realization of the high-frequency mode of electric current $\mathbf{J}_\perp$ stabilization will proceed at considerably lower modulation depth than that observed at the mode of low-frequency stabilization of this current. Thus, the high-frequency mode of the electron current $\mathbf{J}_\perp$ stabilization in SPT may be considered preferable.

V. Conclusions

1) The physical and mathematical model for the electron motion in the decaying magnetic field **B** after the SPT exit plane is developed, taking into account processes of their transfer from the near-cathode zone to the thruster discharge chamber.

2) It is shown that conversion of the electron heat energy into the energy of their flow propagating along the lines of the magnetic field gradient ∇B takes place after the thruster exit plane. Such conversion underlies the mechanism of anomalous electron transfer across field **B** in this discharge zone.

3) Some methods to control electron current $\mathbf{J}_\perp$ in the SPT discharge are defined. One of them is realized by the formation of magnetic field with reduced $\partial B/\partial z$ value in the area of the discharge chamber exit plane in the thruster. Electrons are accumulated in this area because of this, and optimum $\partial n_e/\partial z$ value is reached. Another method suggests formation of a magnetic field with increased curvature of the force lines after the thruster exit plane, in which $\mathbf{J}_\perp$

proportioning is provided, owing to centrifugal force acting on the electrons during their motion along these lines.

4) Electron transfer after the SPT exit plane is unstable and leads to the excitation of at least two types of drift oscillations in plasma: high-frequency magnetohydrogynamic oscillations (∼200 MHz) and low-frequency electromagnetic oscillations (∼20 kHz). Low-frequency drift electromagnetic oscillations develop at the carrier frequency ω of the electron circular rotation during their drift in the crossed electric **E** and magnetic **B** fields.

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